Some PID Notes

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Feedback Control and PID

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Step-by-step instructions?

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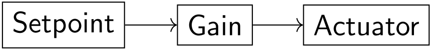
Requires detailed mapping of road.
No way to deal with other cars, ice, weight of car, tire pressure, etc.
You would probably crash!

Feedback Control

This is "open-loop" control:



Only really good if there is low noise or lack of good sensors.
(Actually a good idea for spacecraft.)

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3. Wait until car completes turn
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Robust against changes to car performance and external conditions. You "feedback" on the car's position with your senses.
This works pretty well!

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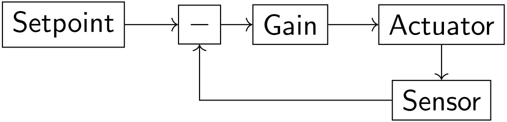

Usually the best (if you can get reliable sensors).

PID Gain

What is gain?

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Many options.
Proportional, Integral, Derivative (PID) gain.
Easy to implement, fairly robust, common

PID Gain

Error: $E(t) = \text{setpoint} - \text{sensor}$.

$$\text{actuator} = P * E(t) + I * \int_0^t E(t')\,dt' + D * \frac{d}{dt}E(t)$$

P, I, D are tunable constants.

PID Gain

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- I: persistent changes, constant drift
- D: avoiding oscillation

PID Gain

Frequency response:
Let $E(t) = \sin(\omega t)$. Then

$$\begin{aligned}\text{actuator} &= P\sin(\omega t) - \frac{I}{\omega}\cos(\omega t) + D\omega\cos(\omega t)\\ &= P\sin(\omega t) + \frac{I}{\omega}\sin\left(\omega t - \frac{\pi}{2}\right) + D\omega * \sin\left(\omega t + \frac{\pi}{2}\right)\end{aligned}$$

PID Gain

Let Δt be the time delay from actuator and sensor, and let the actuator and sensor be linear.
Then (ignore D, D is noisy)

$$\begin{aligned}\text{value} &= P\sin\left(\omega(t-\Delta t)\right) + \frac{I}{\omega}\sin\left(\omega(t-\Delta t) - \frac{\pi}{2}\right) \\ &= P\sin\left(\omega t - \omega\Delta t\right) + \frac{I}{\omega}\sin\left(\omega t - \omega\Delta t - \frac{\pi}{2}\right)\end{aligned}$$

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When $\omega \Delta t + \frac{\pi}{2} = \pi$, I term is negative, and errors are amplified! Need to maintain *phase margin*.
Fortunately, $\frac{I}{\omega}$ decreases, but this limits maximum gain.
Smaller Δt allows higher ω, and thus greater gain.